Resonance in Motion Control Systems

There are no perfectly rigid components in engineering systems. Every component has some flexibility. We can, however, consider the component as “perfectly rigid” and ignore the flexibility, if the effect of that flexibility is not noticeable in the operation or does not effect the output product in a way that matters. Flexibility is the spring effect in mechanical systems. Every moving component in real motion systems can be considered as having at least two properties as a simple model: inertia and stiffness (flexibility). Inertia of the moving machinery coupled with the spring effect of flexibility results in natural frequencies of oscillations in motion (Fig. A-22). Damping in the system is the thrid fundamental conceptual property in that it reflects the resistive force as a function of speed, hence results in loss of energy as function of speed. If the damping is low, the resonance vibrations of very large magnitude occur around the natural frequency (or frequencies).

In servo motion control systems, flexibility becomes a factor that can not be ignored when the bandwidth of the motion gets closer or higher than the frequency of vibrations due to the flexibilty. The source of the problem is mechanical flexibility with low damping.

Examples below shows the kind of vibrations that can occur and the desired solutions by vibration cancelation methods.

1. **The Problem**

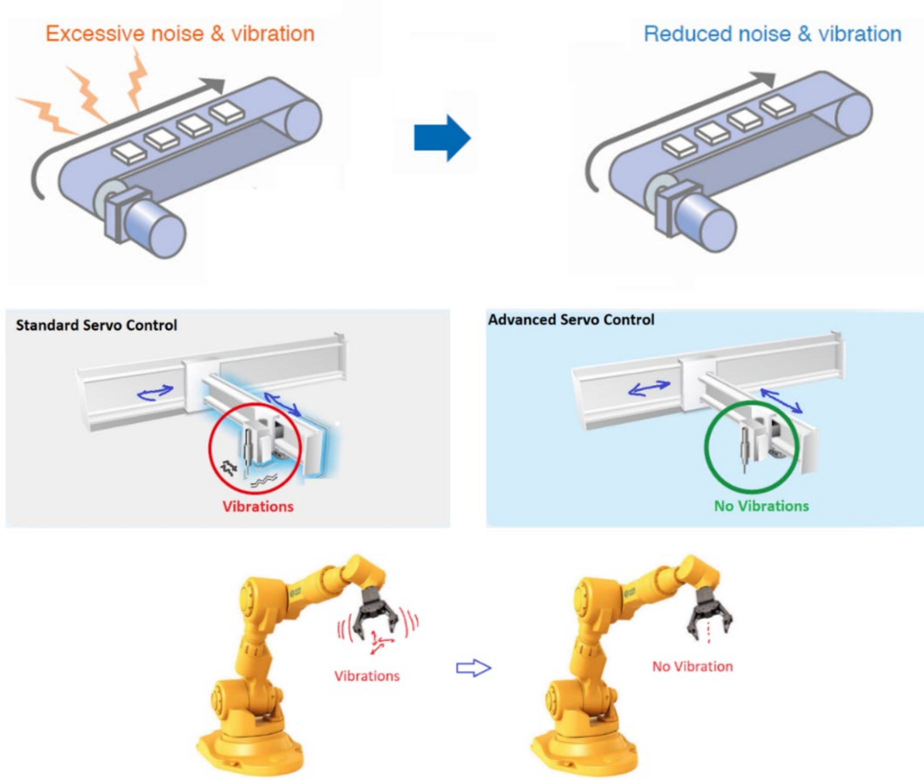


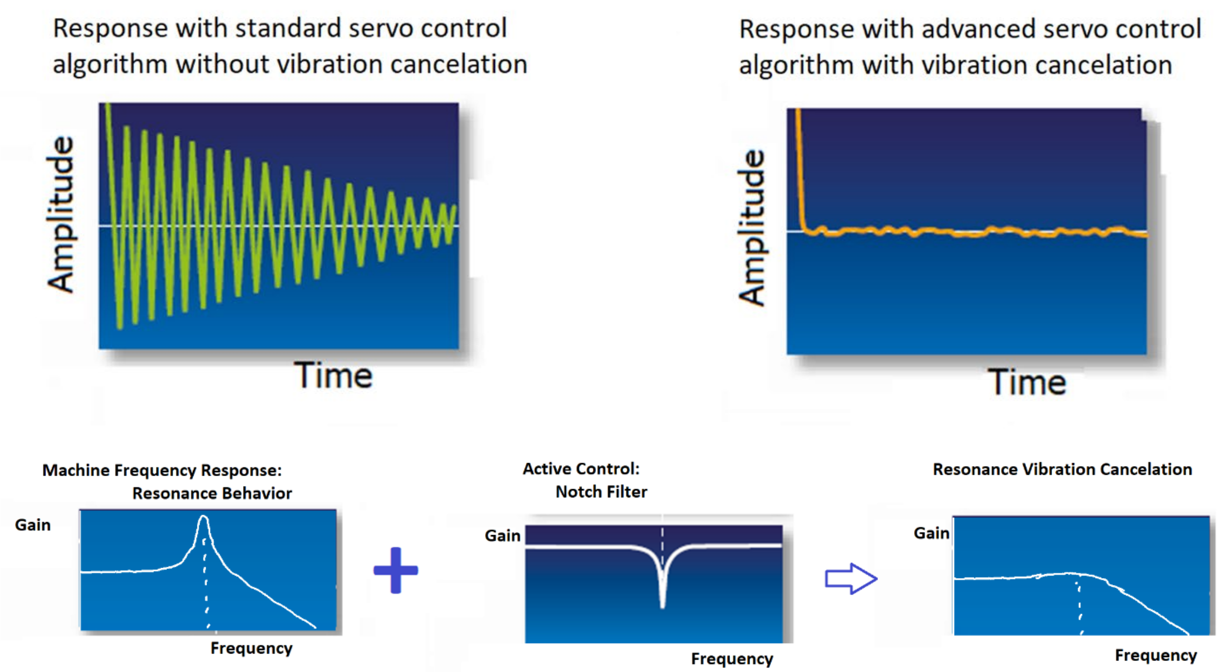
Fig. E-1: Resonant vibrations in motion control systems.

2. Solution Concept: using advanced servo control algorithms eliminate or reduce resonant vibrations; that is implement control logic that is more than standard PID algorithm (Fig. A-23).

Two possible algorithmic solutions are:

1. Introduce notch filter into the control signal output so that the specific resonance frequency is not excited
2. Introduce active damping thru the real time control algorithm at the resonant freqency range.

Option 1 simply does not command motion in the resonant frequency region. Option 2 requires the motion control loop be active at the resonant frequency range and introduce additional damping. Therefore, Option 2 requires a servo control system whose position control bandwidth is as large or larger than the resonant frequency, whereas Option 1 does not require that.

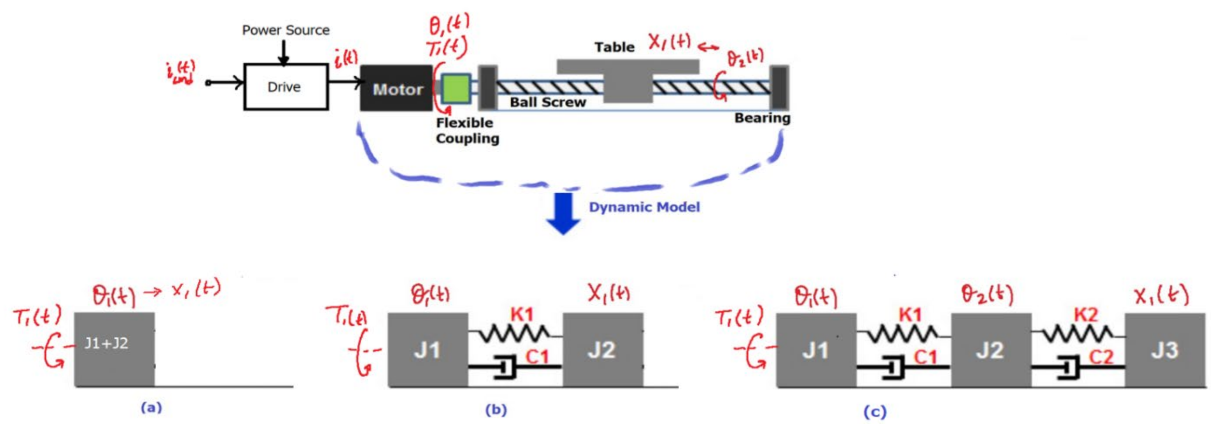


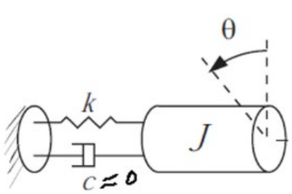
3.The Technical Details of the Solution

Let us consider a translational positioning stage, with servo drive, servo motor, flexible coupling and ball-screw. Depending on the modeling accuracy needed, this can be modeled as (Fig. A-24 and A-25)

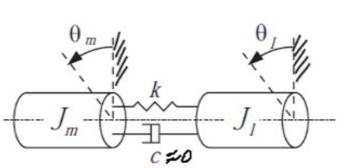
a) a single inertia without flexibility: in this case there is no resonant frequency since there is no flexibility
b) (assumption here is that flexibility is negligible)
c) two inertias (motor side and flexible coupling plus ball-screw side) coupled with flexible element. Due to two inertias couple flexibly to each other, there will be one natural resonant frequency determined by the two inertias and spring constant.
d) three inertia system where we may consider the Table and ball-screw connection flexibility. Similarly, this system will have two natural resonant frequencies.

Case (b) is a good example for us to discuss the flexibility caused resonance frequencies. The same concept applies to Case (c) and even higher order dynamic cases.





$$w_n = \sqrt{\frac{k}{y}}$$

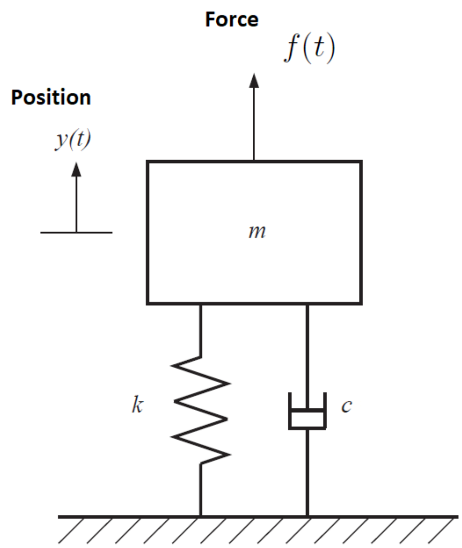


$$w_n = \sqrt{\frac{k}{J_{eff}}}$$

$$J_{eff} = \frac{J_m * J_L}{J_m + J_L}$$

Fig. E-3: Mechanical motion system, its modeling, and natural frequencies.

A second order dynamic system, made of mass, damper, spring and force has the following input-output relationsip. The frequency response of the system (steady state response of position in response to a sinusoidal force input) at different frequencies have the magnitude ratio of output position to input force is shown in the figure below. The natural frequency where largest response occurs is detemined by values of m, c, k and the value of the peak at the resonant frequency is determined by the value of c.



$$m \cdot \ddot{y}(t) + c \cdot \dot{y}(t) + k \cdot y(t) = f(t)$$
$$= k \cdot u(t)$$

$$\frac{Y(s)}{U(s)} = \frac{k}{ms^2 + cs + k}$$
$$= \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2}$$

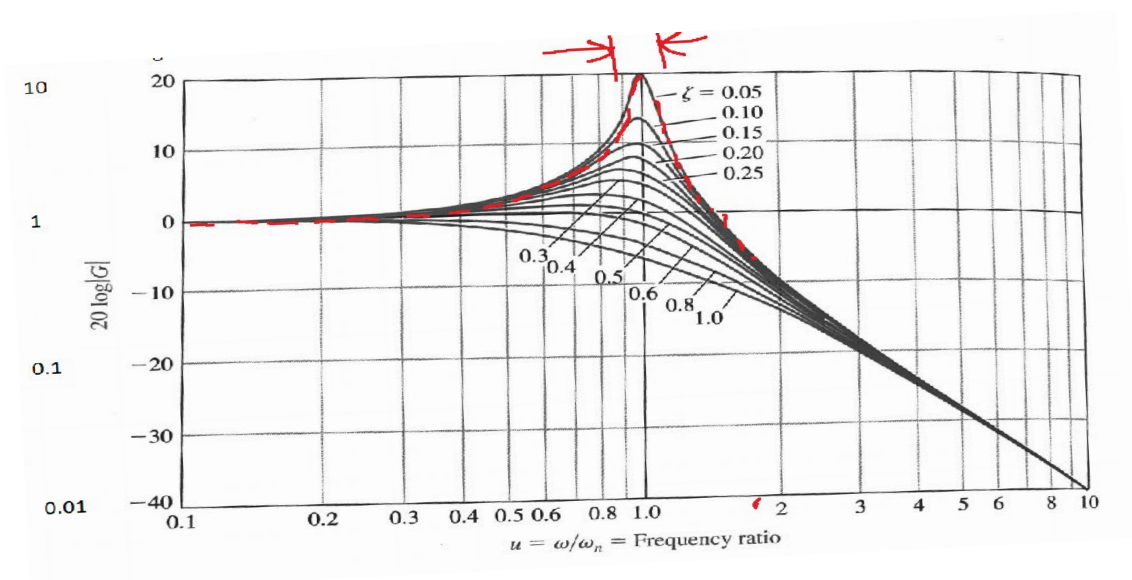


Fig. E-4: Mass-damper-spring system and force input dynamic system.

The dynamic model describing the relationship between motor torque and the angular positions of inertias can be derived using Newton's second law,

Case a:

$$J_G\ddot{\theta}(t) = T(t) - T_d(t)$$

$$\theta(s) = \frac{1}{J_G s^2}(T(s) - T_d(s)$$

Case b:

$$J_m\ddot{\theta}_m(t) + c\left(\dot{\theta}_m(t) - \dot{\theta}_l(t)\right) + k(\theta_m(t) - \theta_l(t) = T_m(t)$$

$$J_l\ddot{\theta}_l(t) + c\left(\dot{\theta}_l(t) - \dot{\theta}_m(t)\right) + k(\theta_l(t) - \theta_m(t) = -T_l(t)$$

where $T_m(t)$ is the motor torque we control, whereas $T_l(t)$ is the disturbance (load) torque, which is not under our control.

Let us look at the transfer function between the motor torque $T_m(t)$ and the motor position and load position. Assume load torque is zero, $T_l(t)$ = 0:0. Assume initial conditions are zero. Using Laplace transforms:

$$(J_m s^2 + cs + k)\theta_m(s) - (cs + k)\theta_l(s) = T_m(s)$$

$$-(cs + k)\theta_m(s) + (J_l s^2 + cs + k)\theta_l(s) = 0$$

The transfer funciton relationship between motor torque and motor position, and that of load position are given by

$$\frac{\theta_m(s)}{T_m(s)} = \frac{1}{J_m}\frac{\left(s^2 + \frac{c}{J_L}s + \frac{k}{J_L}\right)}{s^2(s^2 + \left(\frac{J_m + J_L}{J_m J_L}c\right)s + \left(\frac{J_m + J_L}{J_m J_L}k\right))}$$

$$\frac{\theta_l(s)}{T_m(s)} = \frac{c}{J_m J_L}\frac{s + \frac{k}{c}}{s^2(s^2 + \left(\frac{J_m + J_L}{J_m J_L}c\right)s + \left(\frac{J_m + J_L}{J_m J_L}k\right))}$$

Each transfer function has forth order denominator (which means four poles): one pair of the poles are the rigid body poles at s=0, the other pair is determined by the J_m, J_m, c, k , which is at the resonant frequency.

Motor position transfer function has a second order zero versus the load position transfer function has first order zero. This means that load position transfer function has 90 degree less phase lead than the motor position transfer function, hence it will be go unstable easier than motor position transfer function. As is always the case, if the position control loop is closed using load position only and that the dynamics of the flexible coupling between motor and load can not be neglected, then the closed loop system will be unstable at gains lower than the case it would be for motor position feedback system. Yet, we want to control, hence directly measure, load position quite often. The balance is achieved by using velocity feedback from motor-coupled position sensor and position feedback from load-coupled position sensor.

For example, a ball-screw positioning system behavior can be modeled as a single inertia, neglecting the effect of drive train flexibility (a), or as two seprate inertias connected by a flexible component (b) or three inertias connected by flexible components (c). The degree to which these flexibilities are important is application dependent: if the application bandwidth requirement in position loop is so high (fast) that it is close to the natural frequency due to flexible components, then the flexibility must be taken into account. If the application bandwitdh requirement is slow (much smaller than the natural frequeny, i.e. 1/10 th of it), then the flexibility effect can be ignored.

From resonance concern point of view, the most significant parameter is its stiffness as we discuss below. Inertia of the flexible coupling also effects the resonance frequency since it adds to the total inertia seen by the motor shaft.

The fundamental concern is "mechanical resonance". Any time a moving system involves **inertia** and **flexibility, t**here is a resonance frequency or multiple resonance frequencies. We are interested only in the lowest resonance frequency in general. If we consider a rotoray inertia, J, which is connected to a base with a shaft that has flexibility K (spring like shaft), then there is mechanical resonance freqency

$$w_n = \sqrt{\frac{k}{J}}$$

If the Wn is very large compared to the frequency content of the motion the servo system makes, then there is no concern. However, if the Wn value and the highest frequency the machine will operate, Wm, are closed to each other, then machine may experience resonance ; which is a form of violant oscillations. For example, if the above inertia-spring system was excited by a torque at the same (or very close to) frequency as Wn sinusoidal signal, Fo Sin(Wnt), the displacement magnitude of the inertia would be so large and violent at that same frequency of oscillations that it would destroy itself. This problem can be reduced by increased damping in the system to dissipate the resonant energy, but this is not always possible.

If the motor shaft is connected to another load shaft via a flexible coupling, total inertia would be

$$J = J_{motor} + J_{coupling} + J_{load}$$

Stiffnesses add up lke a parallel resitance, that it, let us assume the stiffness of motor shaft is K_{motor}, stiffness of flexible coupling is $K_{coupling}$ and stiffness of the load shaft is $K_{load\ shaft}$, then the net stifnees K is

$$\frac{1}{K} = \left(\frac{1}{K_{motor}}\right) + \left(\frac{1}{K_{coupling}}\right) + \left(\frac{1}{K_{load\ shaft}}\right)$$

The resulting K will be smaller than the smallest of the three stiffness elements here. Physically, this is easy to understand; the the net flexibility can be larger than the softest link.

No sfaft is perfectly rigid. Every mechanical power transmitting shaft, seemingly rigid, has some flexibility, hence a finite K, albeit be a very large value of K. So, there is always mechanical resonance concern in motion control systems. The design question is to arrange effective inertia, J_{eff}, and stifness K_{eff}, so that resulting Wn is larger than the frequency range the servo motion system will experience. The effect of mechanical resonance can be lessened if there is sufficient damping in the system which turns energy into heat (dissipates energy – damper). However, this may not be possible in every application. Keep in mind large diameter and short length results is higher stiffness of shaft. Shafts connected in “series” acts like “springs in series”, hence the overall stiffness will be less than the smallest stiffness in the link. This is analogous to “paralel resistors” behavior.

Gear reducers effectively reduce the reflected load inertia on the motor shaft, by a factor of $\frac{1}{N^2}$.

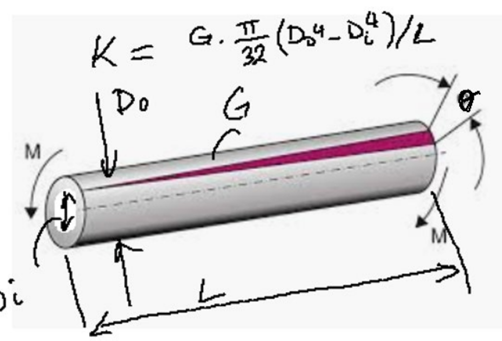
The effect of gear reducer is to change the effective inertia:

$$J_{eff} = J_{motor} + J_{coupling} + (\frac{1}{N^2})\ J_{load}$$

Hence, as we use flexible couplers and gear reducers, we must keep in mind the mechanical resonance. However, just like flexible coupling and solid shafts, gear reducers too introduce some compliance into the motion transmission mechanism, effecting K= K_{eff} . Let us assume that motor shaft is connected to a flexible coupling, then to a gear reducer and then to a load shaft.

$$\frac{1}{K} = (\frac{1}{K_{motor}}) + (\frac{1}{K_{coupling}}) + (\frac{1}{K_{gear}}) + N * (\frac{1}{K_{load\ shaft}})$$

The stiffness after the gear reducer is reflected on the gear reducer, that is the stiffness of the load shaft is scaled by the gear ratio.



G → modulus of rigidity of material.

$D_0, D_i, L \rightarrow geomatric\ diminsions.$

K = torsional stiffness = $\frac{M}{\theta}$

Fig. E-5: A mechanical shaft and the fliexibility it introduces in a rotary motion system.

Notice that for a given shaft material, torsional stiffness increases by power 4 of diameter, and decreases with power 1 of L. For instance, a solid shaft with double (x2) the diameter compared to another shaft of otherwise same properties will be 2^4 = 16 time higher stiffness. Whereas, a shaft with double (x2) the length compated to another shaft with otherwise same properties will have ½ of the stiffness.

Advanced Servo Control Algorithm with Noise Cancelation: PID Algorithm plus Notch Filters

The resonance frequencies can be determined in advance by testing. Then, the resonance effect can be cancelled (damped) by electronic control means in software using so called "anti-resonance filters", provided the actuator components have the necessary bandwidth that is large enough to cover the resonant frequency.

1. By using Notch filters between the command signal and amplification signal (before the current loop)
2. By using Low Pass filters and/or Notch Filters on the command signal (before the position servo loop).

The only challenge is that if the resonance frequency changes during operation, this needs to be detected and anti-resonance filter (i.e., Notch filter) parameters need to be adjusted to exactly target those frequencies.

Cancelling resonance with notch filters is conceptually possible and simple, but in practice it has limitations. It is perfectly possible that we can design a notch filter that targets to cancel the frequency content of the control signal (signal coming out of the servo control algorithm, i.e. output of the PID algorithm) at selected frequency or frequencies of natural resonance.

Two possible locations in the control algorithm where the Notch filter can be inserted are shown below. Location 1 is the input to the servo loop where we "notch" the command signal's freqeuncy content. Locaiton 2 is the input the current loop where we "notch" the frequency content of the position and velocity loop output which is the current loop input. Depending on the applications, we may insert one or more (two, three or more) Notch Filters in both locations, each Notch filter targetting the cancellation of a specific frequency of motion. For instance, if there are three distinct resonance frequencies, we can insert six Notch filters: three at the command input (location 1) and three at the current loop input (location 2). It is also reasobable to insert them only in one location.

In a given motion controller and servo drive combination, we can **enable/disable** Notch filters at the desired locations, as well as **specify the parameters** of each enabled Notch filter to fit the needs the specific application.

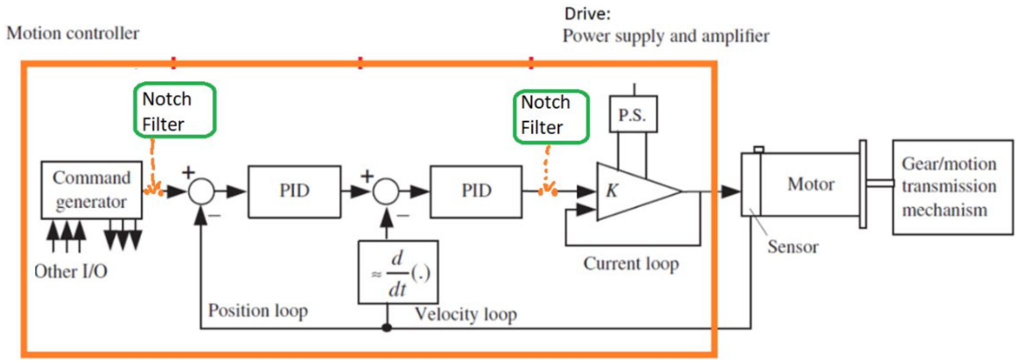


Fig. E-6: Notch filter implementation in servo control algorithm: insert notch filter at the output of the algorithm and/or at the command signal input.

A notch filter has a transfer function like any other filter, as follows, i.e. two zeros and two poles where the damping values of poles and zeros are different.

We select the parameters in order to achieve, gain of unity at all frequencies and very low gain around the resonant frequency. This generally results in parameters that choose for wz wnd wp to be same value , and damping for poles are rather high, whereas damping for zeros are rather low.

Notch Filter (second order)

$$\frac{(s^2/{\omega_z}^2) + 2\xi_z(s/\omega_z) + 1}{(s^2/{\omega_p}^2) + 2\xi_p(s/\omega_p) + 1}$$

Parameters
$\omega_z,\ \omega_p,\ \xi_z,\ \xi_p$

The problem is that in real life conditions, the natual frequencies may change as load and stiffness conditions change over time or even during a normal cycle. As a result, unless we have an algorithmic and sensory way to detect the actual natural frequencies (which is not easy to do reliably), cancellation of natural resonance conditions via notch filters (implemented digitallly as part of PID servo control algorithm) is not practically effective in applications where the resonant frequency (frequencies) is not known accurately or that it (they) may change over time or operating conditions. If we make the notch filter to cover a wide range of frequencies to be able to cover a wide range of possible resonance frequencies, than the notch filter behaves more like a low-pass filter which ends up slowing down the closed loop bandwidth of the system.

In summary, cancellation of resonant frequency (frequencies) can be accomplished effectively and reliably only in applications where we accurately know the resonant frequency (frequencies) or we have a sensing system along with an estimation algorithm to accurately and reliably detect the actual resonant frequencies.

% Matlab file for Notch Filter Frequency Response Plot

```
% Filename: notch1.m
% Notch filter frequency response and step response.

% Select the parameters of the Second Order Notch Filter

  w_z = 250 ;
  w_p = 250 ;
  psi_p = 0.7 ;

  psi_z=  0.05  ;


% Plot Bode plot of the Notch Filter

  num = [1/(w_z^2)      2*psi_z/w_z       1 ] ;
  den  = [1/(w_p^2)      2*psi_p/w_p       1 ] ;

  w= logspace(1,4,200) ;
  [mag,phase]=bode(num,den,w) ;
  mag1 = 20*log10(mag) ;

 subplot(211);
 semilogx(w,mag1) ;
 xlabel('w/w_n'); ylabel('20log|(G(jm))| (dB) ') ; grid on;
 hold on ;

subplot(212);
 semilogx(w,phase) ;
 xlabel('w/w_n'); ylabel('phase') ; grid on;
 hold on ;
```

Fig. A-28: A second order Notch filter frequency response.

Demo: Positioning System with Vibration Suppression

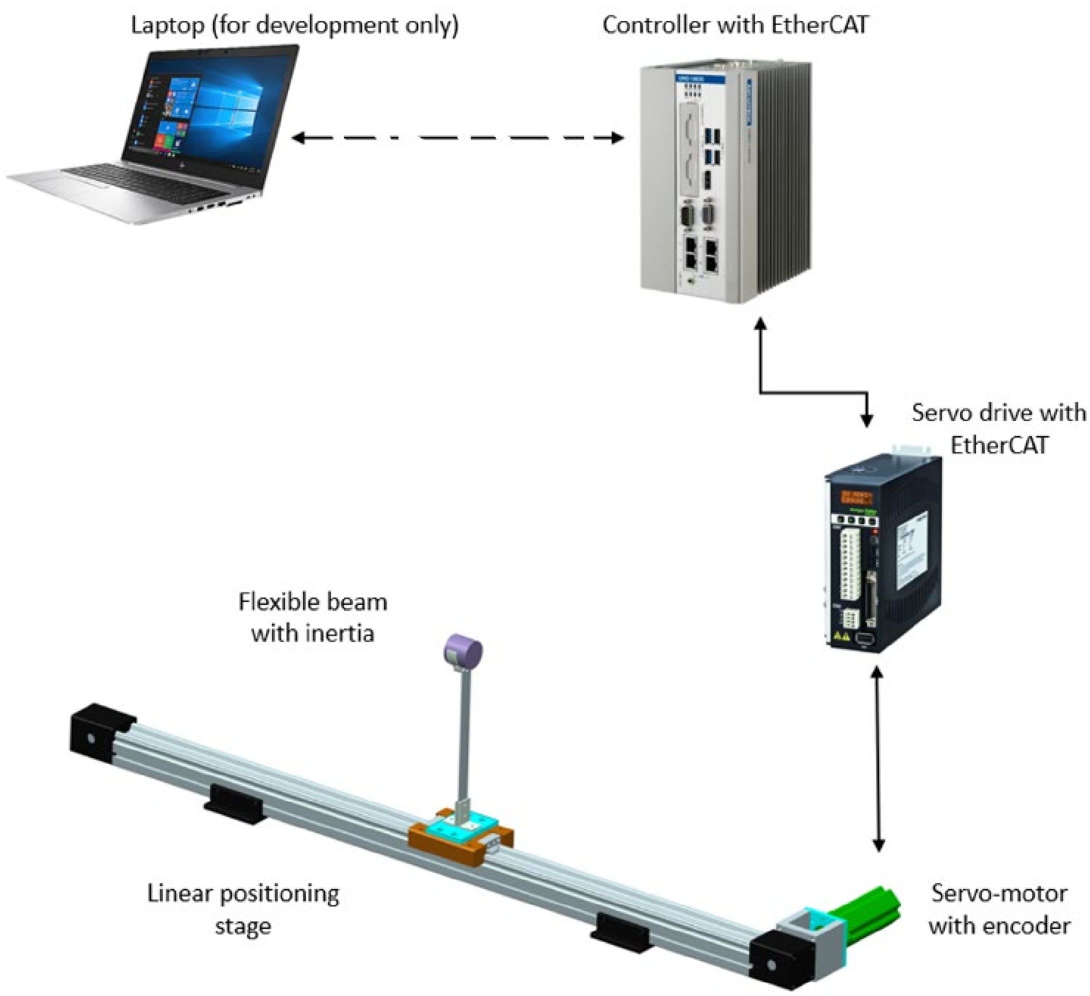


Fig. E-7: Servo positioning system with resonant vibrations and suppression of it via advanced servo control algorithms.

CODESYS code:
Standard PID control plus Notch filter implementation
For each second order Notch filter we have 4 parameters to specify
We can implement 1, 2, 3, or more second order Notch Filters in series to cancel multiple resonant frequencies; one notch filter targeting one resonant frequency

Fig. E-8: Video of a demonstration of resonant vibration suppression via active control: servo control algorithm is standard PID type plus Notch filter.

References

Cetin, S., Khandekar, F., Motion Control Software using CODESYS and EtherCAT, 2022.

Cetinkunt, S., Mechatronics with Experiments, John Wiley and Sons, 2012, Second Edition.

Servo Basics by Yaskawa: https://www.youtube.com/watch?v=Gzo9m0tMD0A&feature=emb_rel_pause

Drive Basic by Yaskawa: https://www.youtube.com/watch?v=3-cs4eEiBWo

Vibration Suppression, by Yaskawa: https://www.youtube.com/watch?v=SstlUdj_xXA